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Alternative Determines Positivity of Hexagonal Fuzzy Numbers and Their Alternative Arithmetic

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Abstract: There are quite a lot of arithmetic operations for hexagonal fuzzy numbers, most of them only define positive fuzzy numbers and few are discussing negative fuzzy numbers. And rarely found inverse of a fuzzy hexagonal number. So, often the results obtained in a hexagonal fuzzy linear equation system are not compatible. In this paper, we will discuss arithmetic alternatives on fuzzy hexagonal numbers. In this paper will definitions of positive and negative fuzzy numbers based on the concept of wide area covered by hexagonal fuzzy numbers in quadrant I and in quadrant II (right and left segments called r). From the concept of positivity and negativity the hexagonal fuzzy numbers will be constructed arithmetic alternatives for hexagonal fuzzy numbers. At the final part be given an inverse for a hexagonal fuzzy number so that, so for any fuzzy number there is an inverse hexagonal fuzzy number and its multiplication produces an identity.

Keywords: Fuzzy Number, Arithmetic Fuzzy Numbers, Hexagonal Fuzzy Numbers

1. Introduction

Fuzzy logic is part of mathematics science introduced by L. A Zadeh in 1965 [8, 9]. Fuzzy set theory permits the gradual assessment of the membership of elements in a set which is decribed in the interval [0, 1]. Fuzzy set theory is applied using the membership functions. Previous researchers defined fuzzy hexagonal with $\tilde{a} = (a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$ and only defined positive fuzzy with results that are not compatible yet [10-13, 15, 16].

One that often appears in various forms of arithmetic for hexagonal fuzzy numbers is for any hexagonal fuzzy number $\tilde{u}$ not necessarily be valid $\tilde{u} - \tilde{u}$ =0 and also not necessarily there $\frac{1}{\tilde{u}}$ so that $\tilde{u} \otimes \frac{1}{\tilde{u}} = \tilde{I}$ like the arithmetic given by the study [10-13, 15-16]. in [13] a positive fuzzy number is defined, if $\tilde{a}_i \geq 0$, for all $i = 1, 2, 3, 4, 5, 6$ and the opposite is said negative this condition does not answer for fuzzy hexagonal numbers contain 0.

While the study [11] discusses in the α-cut form but the hexagon fuzzy number form does not convex and also the conditions mentioned above also do not apply. Other side [15] uses the concept of centroid, but still the above conditions are met and still do not solve the problem for fuzzy numbers that contain 0.

Based on the conditions above, the author feels the need to define the concept of positivity of a fuzzy hexagonal number by using the difference in the concept of wide area in quadrant I with quadrant II (the difference between the area to the right of the r axis and the area to the left of the r axis) and the arithmetic alternative will be constructed for fuzzy hexagonal numbers with convex. In particular the form $\frac{1}{\tilde{u}}$ will also be constructed for any fuzzy number $\tilde{u}$ so that $\tilde{u} \otimes \tilde{v} = \tilde{I}$. While, for multiplication will be described in various cases, so fulfilled, positive fuzzy numbers multiplied by negative fuzzy numbers must be negative, negative fuzzy numbers multiplied by negative fuzzy numbers must be positive and form other cases. The author will define hexagonal fuzzy numbers in the form $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ on condition $a < b$, $\beta \leq \alpha$, $\delta \leq \gamma$ with α distance left from the center a, β distance left from $(a - \alpha)$, γ distance right from the center b and δ distance right from $(b + \gamma)$. By determining alternative fuzzy arithmetic numbers offered. It is expected to solve all matrix problems whose elements are fuzzy hexagonal numbers.

2. Preliminaries

Some basic definition and theories related to fuzzy number has been discussed by [1-7].

Definition 2.1 Fuzzy number is a fuzzy set $\tilde{u}: R \rightarrow [0,1]$ which satisfies the following:

a. $\tilde{u}(x) = 0$ outside the interval $[a-\alpha-\beta, b+\gamma+\delta]$.
b. There exist real number x in interval $[a-\alpha-\beta, b+\gamma+\delta]$ such that,
i. $\tilde{u}(r)$ monotonic increasing in interval $[a-\alpha-\beta, a-\alpha]$ and $[a-\alpha, a]$.
ii. $\tilde{u}(r)$ monotonic decreasing in interval $[b, b+\gamma]$ and $[b+\gamma, b+\gamma+\delta]$.
iii. $\tilde{u}(x) = 1$ for $a \leq x \leq b$.
iv. $\tilde{u}(x) = 0.5$ for $r = a-\alpha$, and $r = b+\gamma$.
v. $\alpha \geq \beta$ and $\gamma \geq \delta$.

Definition 2.2 A Fuzzy number $\tilde{u}$ in R are defined as function pair $\tilde{u} = (P_1(r), Q_1(r), Q_2(r), P_2(r))$ which statisfy the following:

a. $P_1(r)$ is a bounded left continuous non decreasing function over [0, 0.5].
b. $Q_1(r)$ is a bounded left continuous non decreasing function over [0.5, 1].
c. $Q_2(r)$ is a bounded left continuous non increasing function over [1, 0.5].
d. $P_2(r)$ is a bounded left continuous non increasing function over pada [0.5, 1].

Fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4, a_5, a_6$ has been discussed by [3-5, 9, 12] with membership function given as follows :

$$\mu_{\tilde{a}}(x) = \mu_{\tilde{a}}(a_1, a_2, a_3, a_4, a_5, a_6) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & a_2 \leq x \leq a_3 \\ 1, & a_3 \leq x \leq a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \leq x \leq a_6 \\ 0, & x > a_6 \end{cases}$$

The above hexagonal fuzzy number can be written in the parametric form $\tilde{a} = (P_1(r), Q_1(r), Q_2(r), P_2(r))$ with

$$P_1(r) = 2r(a_2 - a_1) + a_2$$

$$Q_1(r) = 2\alpha(a_3 - a_2) - a_3 + 2a_2$$

$$Q_2(r) = -2\alpha(a_5 - a_4) + 2a_5 - a_4$$

$$P_2(r) = -2\alpha(a_6 - a_5) + a_6$$

In this paper, we discuss hexagonal fuzzy number in the form $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ with a and b central points, α distance left from center a, β distance left from center α and γ distance right from center point b, δ distance right from center point γ. The membership functions of hexagonal fuzzy number $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$, that is

$$\mu_{\tilde{u}}(x) = \mu_{\tilde{u}}(a, b, \alpha, \beta, \gamma, \delta) = \begin{cases} \frac{1}{2}-\frac{1}{2}\left(\frac{x-a+\alpha}{\beta}\right), a-\alpha-\beta \leq x \leq a-\alpha \\ 1+\frac{1}{2}\left(\frac{x-a}{\beta}\right), a-\alpha \leq x \leq a \\ 1, a \leq x \leq b \\ 1-\frac{1}{2}\left(\frac{x-b}{\gamma}\right), b \leq x \leq b+\gamma \\ \frac{1}{2}-\frac{1}{2}\left(\frac{x-b-\gamma}{\delta}\right), b+\gamma \leq x \leq b+\gamma+\delta \\ 0, \text{otherwise} \end{cases}$$

Futhermore, the parametric form $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ is $\tilde{u} = (P_1(r), Q_1(r), Q_2(r), P_2(r))$ with:

$$P_1(r) = a - (2-2r)\beta - (\alpha - \beta)$$

$$Q_1(r) = a - (2-2r)\alpha$$

$$Q_2(r) = b + (2-2r)\gamma$$

$$P_2(r) = b + (2-2r)\delta + (\gamma - \delta)$$

3. Positive Fuzzy and Negative Fuzzy Numbers

In this section a new definition will be given to determine a fuzzy number which is said to be positive fuzzy numbers or negative fuzzy numbers wich will be use modify algebra in multiplying two fuzzy numbers.

Fuzzy numbers are said to be positive (negative) if area $\mu_{\tilde{u}}(x) \geq 0$ ($\mu_{\tilde{u}}(x) < 0$) or $R \geq L (R < L)$ seen from the positive x and x negatives as follows:

1. If fuzzy numbers are only in the x-positive line area then it will be positive fuzzy number or $a - \alpha \geq 0$

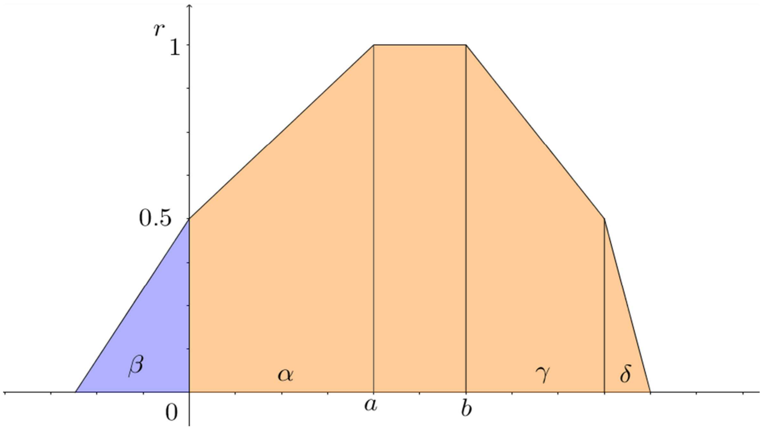

Figure 1. Illustration of positive hexagonal fuzzy numbers.

2. If fuzzy numbers are only in the x-negative line area than it will be negative fuzzy number of $b + \beta \geq 0$

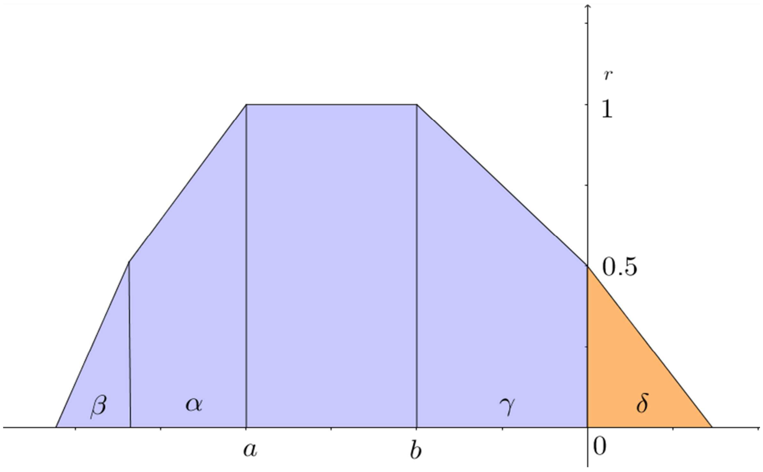

Figure 2. Illustration of negative hexagonal fuzzy numbers.

3. If fuzzy is in both x-positive and x-negative areas, it is divided into 3 cases, as follows:

Case 1 Consider the following figure bellow

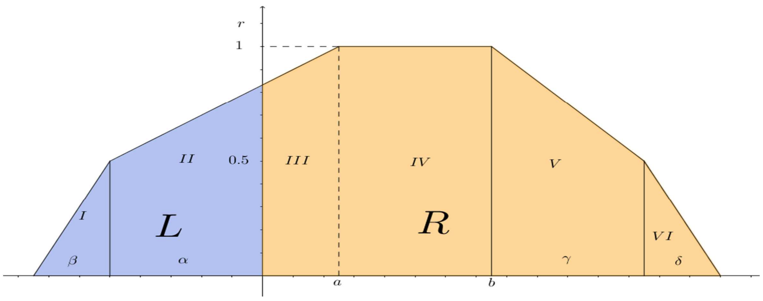

Figure 3. Illustration of case 1 for hexagonal fuzzy numbers.

From the figure above, it can be seen that some areas are at x-positive and others are at x-negative, so they can be divided into 6 (six) area sections, hence

$$L_I = \frac{1}{2}(\beta)\frac{1}{2} = \frac{1}{4}\beta$$

$$L_{II} = \frac{3}{4}(\alpha - a) - \frac{a}{4\alpha}(\alpha - a) = \frac{3}{4}\alpha - a + \frac{a^2}{4\alpha}$$

$$L_{III} = \frac{1}{2}\left(1 - \frac{a}{2\alpha} + 1\right)(\alpha) = a - \frac{a^2}{4\alpha}$$

$$L_{IV} = (b - a)(1) = b - a$$

$$L_V = \frac{1}{2}\left(1+\frac{1}{2}\right)(\gamma) = \frac{3}{4}\gamma$$

$$L_{VI} = \frac{1}{2}\left(\frac{1}{2}\right)(\delta) = \frac{1}{4}\delta$$

Then overall area will be obtained, as follows:

$$R = L_{III} + L_{IV} + L_V + L_{VI} = \frac{\delta + 3\gamma}{4} + b - \frac{a^2}{4\alpha}$$

$$L = L_I + L_{II} = \frac{\beta + 3\gamma}{4} - a + \frac{a^2}{4\alpha}$$

If $a \geq 0, b \geq 0$ and $b + \beta \geq 0$, so $\tilde{u}$ said to be positive fuzzy number if $R - L \geq 0$ or can be written as $\frac{1}{4}\delta + \frac{1}{4}\beta + \frac{3}{4}\gamma - \frac{3}{4}\alpha - b - a - \frac{a^2}{2\alpha} \geq 0$, and $\tilde{u}$ to be said negative if $R - L \leq 0$ or can be written as $\frac{1}{4}\delta + \frac{1}{4}\beta + \frac{3}{4}\gamma - \frac{3}{4}\alpha - b - a - \frac{a^2}{2\alpha} \leq 0$.

Case 2 Consider the following figure bellow

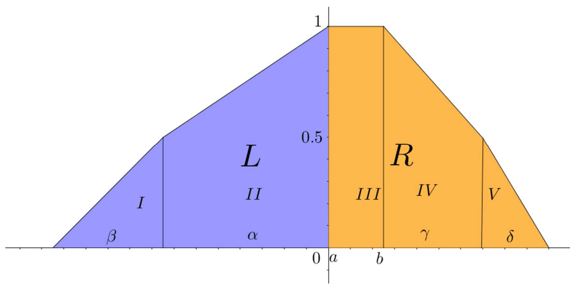

Figure 4. *Illustration of case 2 for hexagonal fuzzy numbers.*

From the figure above, it can be seen that some areas are at x-positive and others are at x-negative, so they can be divided into 5 (five) area sections, hence

$$L_I = \frac{1}{2}(\beta)\frac{1}{2} = \frac{1}{4}\beta$$

$$L_{II} = \frac{1}{2}\left(1+\frac{1}{2}\right)\alpha = \frac{3}{4}\alpha$$

$$L_{III} = b\,(1) = b$$

$$L_{IV} = \frac{1}{2}\left(1+\frac{1}{2}\right)\gamma = \frac{3}{4}\gamma$$

$$L_v = \frac{1}{2}\left(\frac{1}{2}\right)(\delta) = \frac{1}{4}\delta$$

Then overall area will be obtained, as follows:

$$R = L_{III} + L_{IV} + L_V = b + \frac{\delta + 3\gamma}{4}$$

$$L = L_I + L_{II} = \frac{\beta + 3\alpha}{4}$$

If $a \leq 0$ and $b \geq 0$, $\tilde{u}$ said to be positive fuzzy number if $R - L \geq 0$ or can be written as $b + \frac{3}{4}\gamma - \frac{3}{4}\alpha + \frac{1}{4}\delta - \frac{1}{4}\beta \geq 0$ and $\tilde{u}$ to be said negative if $R - L \leq 0$ or can be written as $b + \frac{3}{4}\gamma - \frac{3}{4}\alpha + \frac{1}{4}\delta - \frac{1}{4}\beta \leq 0$.

Case 3 Consider the following figure bellow

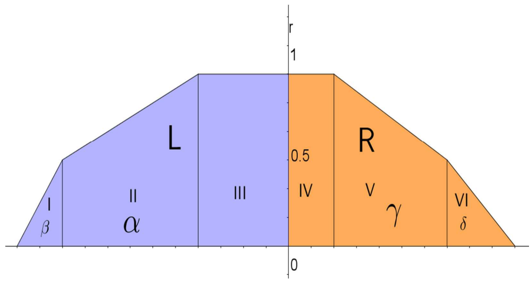

Figure 5. *Illustration of case 3 for hexagonal fuzzy numbers.*

From the figure above, it can be seen that some areas are at x-positive and others are at x-negative, so they can be divided into 5 (sive) area sections, hence

$$L_I = \frac{1}{2}(\beta)\frac{1}{2} = \frac{1}{4}\beta$$

$$L_{II} = \frac{1}{2}\left(1+\frac{1}{2}\right)\alpha = \frac{3}{4}\alpha$$

$$L_{III} = -a\,(1) = -a$$

$$L_{IV} = b\,(1) = b$$

$$L_V = \frac{1}{2}\left(1+\frac{1}{2}\right)\gamma = \frac{3}{4}\gamma$$

$$L_{VI} = \frac{1}{2}\left(\frac{1}{2}\right)(\delta) = \frac{1}{4}\delta$$

Then overall area will be obtained, as follows:

$$R = L_{IV} + L_V + L_{VI} = b + \frac{3\gamma}{4} + \frac{\delta}{4}$$

$$L = L_I + L_{II} + L_{III} = -a + \frac{3\alpha}{4} + \frac{\beta}{4}$$

If $a \leq 0$ and $b \geq 0$, $\tilde{u}$ said to be positive fuzzy number if $R - L \geq 0$ or can be written as $b + a + \frac{3}{4}\gamma - \frac{3}{4}\alpha + \frac{1}{4}\delta - \frac{1}{4}\beta \geq 0$ and $\tilde{u}$ to be said negative if $R - L \leq 0$ or can be written as $b + a + \frac{3}{4}\gamma - \frac{3}{4}\alpha + \frac{1}{4}\delta - \frac{1}{4}\beta \leq 0$.

Case 4 Consider the following figure bellow

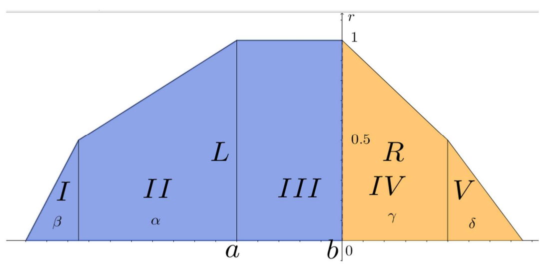

Figure 6. *Illustration of case 4 for hexagonal fuzzy numbers.*

From the figure above, it can be seen that some areas are at x-positive and others are at x-negative, so they can be divided into 5 (five) area sections, hence

$$L_I = \frac{1}{2}(\beta)\frac{1}{2} = \frac{1}{4}\beta$$

$$L_{II} = \frac{1}{2}\left(1 + \frac{1}{2}\right)\alpha = \frac{3}{4}\alpha$$

$$L_{III} = -a(1) = -a$$

$$L_{IV} = \frac{1}{2}\left(1 + \frac{1}{2}\right)\gamma = \frac{3}{4}\gamma$$

$$L_V = \frac{1}{2}\left(\frac{1}{2}\right)(\delta) = \frac{1}{4}\delta$$

Then overall area will be obtained, as follows:

$$R = L_{IV} + L_V = \frac{3\gamma}{4} + \frac{\delta}{4}$$

$$L = L_I + L_{II} = -a + \frac{3\alpha}{4} + \frac{\beta}{4}$$

If $a \geq 0$ and $b \geq 0$, $\tilde{u}$ said to be positive fuzzy number if $R - L \geq 0$ or can be written as $a + \frac{3}{4}\gamma - \frac{3}{4}\alpha + \frac{1}{4}\delta - \frac{1}{4}\beta \geq 0$, and $\tilde{u}$ to be said negative if $R - L \leq 0$ or can be written as $a + \frac{3}{4}\gamma - \frac{3}{4}\alpha + \frac{1}{4}\delta - \frac{1}{4}\beta \leq 0$.

4. New Arithmetic Hexagonal Fuzzy Number

After defining positive fuzzy numbers and negative fuzzy numbers and operation of fuzzy numbers then it will be applied to the multiplication of two fuzzy numbers will be explained in the following below:

Arithmetic algebra operation will be for hexagonal fuzzy numbers. For $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ and $\tilde{v} = (c, d, \theta, \omega, \tau, \sigma)$, then the parametric forms are as follows:

$$\tilde{u} = (P_1(r), Q_1(r), Q_2(r), P_2(r))$$

$$P_1(r) = a - (2 - 2r)\beta - (\alpha - \beta)$$

$$Q_1(r) = a - (2 - 2r)\alpha$$

$$Q_2(r) = b + (2 - 2r)\gamma$$

$$P_2(r) = b + (2 - 2r)\delta + (\gamma - \delta)$$

and,

$$\tilde{v} = (K_1(r), L_1(r), L_2(r), K_2(r))$$

$$K_1(r) = c - (2 - 2r)\omega - (\theta - \omega)$$

$$L_1(r) = c - (2 - 2r)\theta$$

$$L_2(r) = d + (2 - 2r)\tau$$

$$K_2(r) = d + (2 - 2r)\sigma + (\tau - \sigma)$$

Definition 4.1 Let $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ and $\tilde{v} = (c, d, \theta, \omega, \tau, \sigma)$ be two hexagonal fuzzy numbers, then

a. Addition

For $r \in [0, 0.5]$

$$\tilde{u} \oplus \tilde{v} = [P_1(r) + K_1(r), P_2(r) + K_2(r)]$$

$$= [(a - (2 - 2r)\beta - (\alpha - \beta)) + (c - (2 - 2r)\omega - (\theta - w)), (b + (2 - 2r)\delta + (\gamma - \delta) + (d + (2 - 2r)\sigma + (\tau - \sigma))] \quad (1)$$

$$= [a + c - (2 - 2r)(\beta + w) - (\alpha - \beta) - (\theta - \omega), b + d + (2 - 2r)(\delta + \sigma) + (\tau - \sigma)] \quad (2)$$

For $r \in [0.5, 1]$

$$\tilde{u} \oplus \tilde{v} = [Q_1(r) + L_1(r), Q_2(r) + L_2(r)]$$

$$= [a - (2 - 2r)\alpha + c - (2 - 2r)\theta, b + (2 - 2r)\gamma + d + (2 - 2r)\tau] \quad (3)$$

$$= [a + c - (2 - 2r)(\alpha + \theta), b + d + (2 - 2r)(\gamma + \tau)] \quad (4)$$

Transforming back into the hexagonal form, from equation (1) (2) (3) and (4) we have:

$$\tilde{u} \oplus \tilde{v} = (a + c), (b + d), (\alpha + \theta), (\beta + \omega), (\gamma + \tau), (\delta + \sigma)$$

b. Subtraction

For $r \in [0, 0.5]$

$$\tilde{u} \ominus \tilde{v} = [P_1(r) - K_2(r), P_2(r) - K_1(r)]$$

$$= [(a - (2 - 2r)\beta - (\alpha - \beta) - b + (2 - 2r)\sigma + (\tau - \sigma), b + (2 - 2r)\delta + (\gamma - \delta) - c - (2 - 2r)\omega - (\theta - \omega) \quad (5)$$

$$= [a - d - (2 - 2r)(\beta + \sigma) - (\alpha - \beta) + (\tau - \sigma), b - c + (2 - 2r)(\delta + \omega) + (\gamma - \delta) - \theta - \omega) \quad (6)$$

For $r \in [0.5, 1]$

$$\tilde{u} \ominus \tilde{v} = [Q_1(r) - L_2(r), Q_2(r) - L_1(r)]$$

$$= [a - (2 - 2r)\alpha - d + (2 - 2r)\tau, b + (2 - 2r)\gamma - c - (2 - 2r)\theta] \quad (7)$$

$$= [a - d - (2 - 2r)(\alpha + \tau), b - c + (2 - 2r)(\gamma + \theta)] \quad (8)$$

Transforming back into the hexagonal form, from equation (5) (6) (7) and (8) we have:

$$\tilde{u} \ominus \tilde{v} = (a - d), (b - c), (\alpha + \tau)(\beta + \sigma), (\gamma + \theta), (\delta + \omega)$$

c. Scalar product

$$\lambda \otimes \tilde{u} = \lambda \otimes (a, b, \alpha, \beta, \gamma, \delta)$$

$$= \begin{cases} (\lambda a, \lambda b, \lambda \alpha, \lambda \beta, \lambda \gamma, \lambda \delta), \lambda \geq 0 \\ (\lambda b, \lambda a, -\lambda \delta, -\lambda \gamma, -\lambda \alpha, -\lambda \beta)\ \lambda \leq 0 \end{cases}$$

d. Multiplication

If $\tilde{u} = [P_1(r), Q_1(r), Q_2(r), P_2(r)]$ and $\tilde{v} = [K_1(r), L_1(r), L_2(r), K_2(r)]$ are two positive fuzzy numbers, then $\tilde{z} = [\tilde{u} \otimes \tilde{v}]$ for every $r \in [0, 0.5]$ and $r \in [0.5, 1]$. The following is given some cases for hexagonal fuzzy numbers to multiplication operastion.

i. If $\tilde{u}$ is positive and $\tilde{v}$ is positive, then:

For $r \in [0, 0.5]$

$$\begin{cases}\underline{z}(r) = P_1(r)K_1(0.5) + P_1(0.5)K_1(r) - P_1(0.5)K_1(0.5)\\ \overline{z}(r) = P_2(r)K_2(0.5) + P_2(0.5)K_2(r) - P_2(0.5)K_2(0.5)\end{cases}$$

$$\underline{z}(r) = ac - (2-2r)(\beta c - \beta\theta + a\omega - \alpha\omega) - \alpha c - \alpha\theta + \beta c - \beta\theta - a\theta - a\omega + \alpha\omega \quad (9)$$

$$\overline{z}(r) = bd + (2-2r)(\sigma b + \sigma\gamma)(\delta d + \delta\tau + \sigma b + \sigma\gamma) + \gamma d + \gamma\tau - \delta d - \delta\tau + b\tau - \sigma b - \delta\gamma \quad (10)$$

For $r \in [0.5, 1]$

$$\begin{cases}\underline{z}(r) = Q_1(r)L_1(1) + Q_1(1)L_1(r) - Q_1(1)L_1(1)\\ \overline{z}(r) = Q_2(r)L_2(1) + Q_2(1)L_2(r) - Q_2(1)L_2(1)\end{cases}$$

$$\underline{z}(r) = ac - (2-2r)(\alpha c + a\theta) \quad (11)$$

$$\overline{z}(r) = bd + (2-2r)(\gamma d + \tau b) \quad (12)$$

Transforming back into the hexagonal form, from equation (9) (10) (11) and (12) multiplication $\tilde{u}$ positive and $\tilde{v}$ positive can be written as

$$\tilde{z} = \tilde{u} \otimes \tilde{v} = [ac, bd, (\alpha c + a\theta),$$

$$(\beta c - \beta\theta + a\omega - \alpha\omega), (\gamma d + \tau b), (\delta d + \delta\tau + \sigma b + \sigma\gamma)]$$

ii. If $\tilde{u}$ is positive and $\tilde{v}$ is negative, then:
For $r \in [0, 0.5]$

$$\begin{cases}\underline{z}(r) = P_2(r)K_1(0.5) + P_2(0.5)K_1(r) - P_2(0.5)K_1(0.5)\\ \overline{z}(r) = P_1(r)K_2(0.5) + P_1(0.5)K_2(r) - P_1(0.5)K_2(0.5)\end{cases}$$

$$\underline{z}(r) = bc - (2-2r)(-c\delta + \theta\delta + b\omega + \omega\gamma) + \gamma c - \gamma\theta - \delta c + \gamma\theta - b\theta + \omega b + \omega\gamma \quad (13)$$

$$\overline{z}(r) = ad + (2-2r)(-\beta d - \beta\tau + a\sigma - \alpha\sigma) - \alpha d - \alpha\tau + \beta d + \beta\tau + a\tau - a\sigma + \alpha\sigma \quad (14)$$

For $r \in [0.5, 1]$

$$\begin{cases}\underline{z}(r) = Q_2(r)L_1(1) + Q_2(1)L_1(r) - Q_2(1)L_1(1)\\ \overline{z}(r) = Q_1(r)L_2(1) + Q_1(1)L_2(r) - Q_1(1)L_2(1)\end{cases}$$

$$\underline{z}(r) = bc - (2-2r)(-\gamma c + b\theta) \quad (15)$$

$$\overline{z}(r) = ad + (2-2r)(-\alpha d + a\tau) \quad (16)$$

Transforming back into the hexagonal form, from equation (13) (14) (15) and (16) multiplication $\tilde{u}$ positive and $\tilde{v}$ negative can be written as

$$\tilde{z} = \tilde{u} \otimes \tilde{v} = [bc, ad, (b\theta - \gamma c), (-c\delta + \theta\delta + b\omega + \gamma\omega),$$

$$(a\tau - \alpha d\,), (\beta d - \beta\tau + a\sigma - \alpha\sigma)$$

iii. If $\tilde{u}$ is negative and $\tilde{v}$ is positive, then:
For $r \in [0, 0.5]$

$$\begin{cases}\underline{z}(r) = P_1(r)K_2(0.5) + P_1(0.5)K_2(r) - P_1(0.5)K_2(0.5)\\ \overline{z}(r) = P_2(r)K_1(0.5) + P_2(0.5)K_1(r) - P_2(0.5)K_1(0.5)\end{cases}$$

$$\underline{z}(r) = ad - (2-2r)(d\beta + \beta\tau + \alpha\sigma - a\sigma) - \alpha d - \alpha\tau + d\beta + \tau\beta + a\tau - a\sigma + \alpha\sigma \quad (17)$$

$$\overline{z}(r) = bc + (2-2r)(\delta c - \delta\theta - b\omega - \delta\omega) + \gamma c - \gamma\theta - \delta c + \delta\theta + \omega b + \omega\gamma \quad (18)$$

For $r \in [0.5, 1]$

$$\begin{cases}\underline{z}(r) = Q_1(r)L_2(1) + Q_1(1)L_2(r) - Q_1(1)L_2(1)\\ \overline{z}(r) = Q_2(r)L_1(1) + Q_2(1)L_1(r) - Q_2(1)L_1(1)\end{cases}$$

$$\underline{z}(r) = ad - (2-2r)(a\tau - \alpha d) \quad (19)$$

$$\overline{z}(r) = bc + (2-2r)(\gamma c - b\theta) \quad (20)$$

Transforming back into the hexagonal form, from equation (17) (18) (19) and (20) multiplication $\tilde{u}$ positive and $\tilde{v}$ negative can be written as

$$\tilde{z} = \tilde{u} \otimes \tilde{v} = [ad, bc, (\alpha d - a\tau), (\beta d + \beta\tau + \alpha\sigma - a\sigma),$$

$$(\gamma c - b\theta), (\delta c - \delta\theta - b\omega - \gamma\omega)]$$

iv. If $\tilde{u}$ is negative and $\tilde{v}$ is negative, then:
For $r \in [0, 0.5]$

$$\begin{cases}\underline{z}(r) = P_2(r)K_2(0.5) + P_2(0.5)K_2(r) - P_2(0.5)K_2(0.5)\\ \overline{z}(r) = P_1(r)K_1(0.5) + P_1(0.5)K_1(r) - P_1(0.5)K_1(0.5)\end{cases}$$

$$\underline{z}(r) = bd - (2-2r)(-\delta d - \delta\tau - \sigma b - \sigma\gamma) - \gamma d - \gamma\tau + \delta d - \delta\tau - b\tau + \sigma b + \delta\gamma \quad (21)$$

$$\overline{z}(r) = ac + (2-2r)(-\beta c + \beta\theta - a\omega - \alpha\omega) - \alpha c - \alpha\theta + \beta c + \beta\theta + a\theta - a\omega + \alpha\omega \quad (22)$$

For $r \in [0.5, 1]$

$$\begin{cases}\underline{z}(r) = Q_2(r)L_2(1) + Q_2(1)L_2(r) - Q_2(1)L_2(1)\\ \overline{z}(r) = Q_1(r)L_1(1) + Q_1(1)L_1(r) - Q_1(1)L_1(1)\end{cases}$$

$$\underline{z}(r) = bd - (2-2r)(-\gamma d - \tau b) \quad (23)$$

$$\overline{z}(r) = ac + (2-2r)(-\alpha c - a\theta) \quad (24)$$

Transforming back into the hexagonal form, from equation (21) (22) (23) and (24) multiplication $\tilde{u}$ negative and $\tilde{v}$ negative can be written as

$$\tilde{z} = \tilde{u}_r \otimes \tilde{v}_r = [bd, ac, (-\gamma d - \tau b),$$

$$-(\delta d + \delta\tau + \sigma b + \sigma\gamma), (-\alpha c - \alpha\theta),$$

$$-(\beta c - \beta\theta + a\omega - \alpha\omega)]$$

e. Invers
The identity element for hexagonal fuzzy numbers is:

$$\tilde{I} = (1, 1, 0, 0, 0, 0)$$

Where $\tilde{I} = (1, 1, 0, 0, 0, 0)$ positive. Let two fuzzy number $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ and $\tilde{v} = (c, d, \theta, \omega, \tau, \sigma)$ have invers:

$$\tilde{v} = \frac{1}{\tilde{u}} = u^{-1}$$

Will be indicated $\tilde{u} \otimes \tilde{v} = (1, 1, 0, 0, 0, 0)$. Fuzzy number $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$ the condition for having invers is $a, b \neq 0$. Therefore, inverse forhexagonal number consist of two cases, as follows:

(i) $\tilde{u} > 0$ and $\tilde{v} > 0$,

$$\tilde{u} \otimes \tilde{v} = (a, b, \alpha, \beta, \gamma, \delta) \otimes (c, d, \theta, \omega, \tau, \sigma)$$

$$= \big(ac, bd, (\alpha c + \alpha\theta), (\beta c - \beta\theta + a\omega - \alpha\omega), (\gamma d + \tau b), (\delta d + \delta\tau + \sigma b + \sigma\gamma)\big) = (1, 1, 0, 0, 0, 0).$$

(ii) $\tilde{u} < 0$ and $\tilde{v} < 0$,

$$\tilde{u} \otimes \tilde{v} = (a, b, \alpha, \beta, \gamma, \delta) \otimes (c, d, \theta, \omega, \tau, \sigma)$$

$$= bd, ac, (-\gamma d - \tau b), -(\delta d + \delta\tau + \sigma b + \sigma\gamma), (-\alpha c - \alpha\theta), -(\beta c - \beta\theta + a\omega - \alpha\omega) = (1, 1, 0, 0, 0, 0).$$

So the inverse for hexagonal fuzzy numbers is:

$$\tilde{v} = \frac{1}{\tilde{u}} = u^{-1} = \begin{pmatrix} \frac{1}{a}, \frac{1}{b}, -\frac{\alpha}{a^2}, -\left(\frac{\alpha\beta}{a^3 - a^2\alpha} - \frac{\beta}{a^2 - a\alpha}\right), \\ -\frac{\gamma}{b^2}, -\left(\frac{\delta}{b^2 + b\gamma} + \frac{\gamma\delta}{b^3 + b^2\gamma}\right) \end{pmatrix}$$

5. Conclusion

In this paper the author defines a new form of the hexagonal fuzzy number $\tilde{u} = (a, b, \alpha, \beta, \gamma, \delta)$. Furthermore, the authors define positive hexagonal fuzzy numbers and negative fuzzy numbers, so can determine the multiplication of two fuzzy numbers, which will produce compatible results. The next discussion can develop hexagonal fuzzy using the Cramer, Gauss-Jacobi and others methods.

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